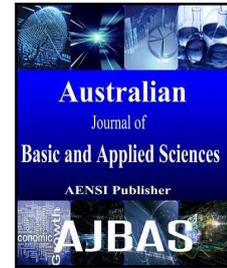




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Stability Analysis and Qualitative Analysis of Chickenpox Model with Sanitation Campaign

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ABSTRACT

In this study, we proposed and analyzed a mathematical model to study the dynamics transmission of chickenpox with effect of sanitation campaign. The model is analyzed using stability theory of differential equations and computer simulations. The results showed that there were two equilibrium points: Chickenpox-free equilibrium point and Chickenpox - present equilibrium point. The qualitative analysis results depend on the Chickenpox basic reproductive number (R_0). We derived the Chickenpox reproductive number by using the next generation method. Stabilities of the model are determined by Routh-Hurwitz conditions. If $R_0 < 1$, then the Chickenpox-free equilibrium point is local asymptotically stable, but if $R_0 > 1$, then the Chickenpox-present equilibrium point is local asymptotically stable. The graphical representations are provided to qualitatively support the analytical results. It concluded that with an increase in the effectiveness of individuals' sanitation, the number of infected human will be decreased.

INTRODUCTION

Chickenpox is a contagious viral disease caused by the Varicella-Zoster Virus (VZV). It causes a blister-like rash, itching, tiredness, and fever. Chickenpox can be serious, especially in babies, adults, and people with weakened immune systems. It spreads easily from infected people to others who have never had chickenpox or received the chickenpox vaccine. Chickenpox spreads in the air through coughing or sneezing. It can also be spread by touching or breathing in the virus particles that come from chickenpox blisters. Chickenpox most commonly causes an illness that lasts about 5-10 days. The symptom of chickenpox is a rash that turns into itchy, fluid-filled blisters that eventually turn into scabs. The rash may first show up on the face, chest, and back then spread to the rest of the body, including inside the mouth, eyelids, or genital area. It usually takes about one week for all the blisters to become scabs (CDC,2015). From 1 January 2015 - 15 November 2015, a total of 47,407 cases were report from 77 provinces in Thailand (BOE.2015).

Mathematical models have become an important tool for understanding the spread and control of disease. Esson *et al.* (2014) proposed and analyzed the SEIR model of chickenpox in Agona West Municipality of Ghana. The study showed that the basic reproductive number with equal two. The spread of chickenpox can be prevented by reducing the rate at which people are exposed to the disease. Sensitivity analysis of SEIR model showed that the latency rate was more sensitive to the model than the transmission rate and the recovery rate. Yang, Chen and Zhang. (2013) proposed the model to consider the spread of Hand Foot Mouth Disease

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(HFMD) in Mainland China. The results showed that the spread of HFMD is best controlled by increasing personal protect against the disease. So the objective of this research is to investigate the effect of sanitation campaign on the transmission model of chickenpox, which the diagram of chickenpox transmission as shown in Fig. 1.

The organization of this paper is as follows: In section 2, we formulate the transmission model of chickenpox. In section 3, present the model analysis with regard to equilibrium points and its stabilities and Chickenpox reproductive number, In section 4, we present the numerical results for two states, at Chickenpox-free equilibrium point and Chicken- present equilibrium point. And in the last section, we present the conclusion.

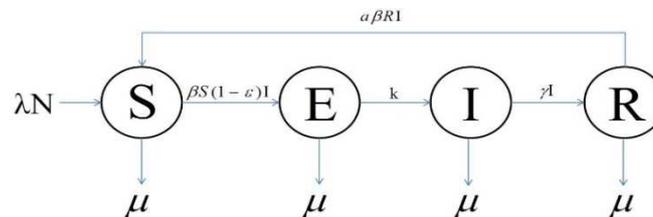


Fig. 1: Diagram of chickenpox transmission.

$$\frac{dS}{dt} = \lambda N + a\beta RI - \beta S(1-\varepsilon)I - \mu S \quad (1)$$

$$\frac{dE}{dt} = \beta S(1-\varepsilon)I - kE - \mu E \quad (2)$$

$$\frac{dI}{dt} = kE - \gamma I - \mu I \quad (3)$$

$$\frac{dR}{dt} = \gamma I - \mu R - a\beta RI \quad (4)$$

$$\text{with } S+E+I+R=N \quad (5)$$

The dynamical model can be represented by the following system of differential equations as follows,

S is the number of susceptible individuals at time t ,

E is the number of exposed individuals at time t ,

I is the number of infectious individuals at time t ,

R is the number of recovered individuals at time t ,

λ is the birth rate of individuals,

μ is the natural death rate of individuals,

β is the contact rate,

k is the incubation period,

γ is the recovery rate,

a is re infected induce loss recovery,

ε is the effectiveness of individuals sanitation,

N is the total number of individuals population.

From, $N = S + E + I + R$

Consider

$$\frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = \lambda N + a\beta\beta - \beta S(1-\varepsilon)I - \mu S + \beta S(1-\varepsilon)I - kE - \mu E$$

$$+ kE - \gamma I - \mu I + \gamma I - \mu R - a\beta\beta R$$

$$= \lambda N - \mu S - \mu E - \mu I - \mu R$$

$$= \lambda N - \mu(S + E + I + R)$$

$$= \lambda N - \mu N$$

$$\frac{d(S + E + I + R)}{dt} = (\lambda - \mu)N$$

$$\frac{dN}{dt} = (\lambda - \mu)N$$

Thus, $\frac{dN}{dt} = 0$, this means that the population is constant.

3 Model analysis:

Equilibrium Points:

By using the standard method for analyzing our model, this system has two equilibrium points; Chicken pox –free equilibrium point and Chickenpox- present equilibrium point. We obtained these by setting the right hand side of equations, (1)-(4) to zero. Doing this, we obtain

1. Chickenpox - Free Equilibrium:

(CFE) denoted by $E_0(S, I, R, B)$

In the case of the absence of the disease, that is $I = 0$. We obtained $S = N$ $E = 0$ $R = 0$. Thus,

$$E_0(S, E, I, R) = E_0(N, 0, 0, 0).$$

Chickenpox - Present Equilibrium:

(CPE) denoted by $E_1(S^*, E^*, I^*, R^*)$

In the case where the disease is present, that is $I \neq 0$. We obtained

$$S^* = \frac{\lambda N + a\beta \left(\frac{\gamma^*}{\mu + a\beta I^*} \right) I^*}{\mu + \beta(1-\varepsilon)I^*}, E^* = \frac{\beta \left(\lambda N + a\beta \left(\frac{\gamma^*}{\mu + a\beta I^*} \right) I^* \right) (1-\varepsilon) I^*}{(k+\mu)\mu + (k+\mu)\beta(1-\varepsilon)I^*}, I^* = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, R^* = \frac{\gamma^*}{\mu + a\beta I^*} \quad (6)$$

Where

$$a = a\beta(\gamma + \mu)(k + \mu)\beta(1 - \varepsilon) - (1 - \varepsilon)a\beta^2k\gamma$$

$$b = a\beta(\gamma + \mu)(k - \mu)\mu - a\beta(1 - \varepsilon)\beta k\lambda N + \mu(\gamma + \mu)(k + \mu)\beta(1 - \varepsilon)$$

$$c = \mu(\gamma + \mu)(k + \mu)\mu - \mu(1 - \varepsilon)\beta k\lambda N$$

Chickenpox Reproductive Number:

The Chickenpox reproductive number (R_0) (threshold condition in epidemiology) is the number of secondary infections induced by an infected individual introduced into the total susceptible population (Anderson and May, 1991). By using the next generation method and used spectral radius (Van den Driessche and Watmough, 2002). Doing this, we rewritten the system in matrix form.

$$\frac{dx}{dt} = F(x) - V(x), x = [S, E, I, R]^T$$

$$F(x) = \begin{bmatrix} 0 \\ \beta S(1-\varepsilon)I \\ 0 \\ 0 \end{bmatrix} \text{ and } V(x) = \begin{bmatrix} -\lambda N - a\beta RI + \beta S(1-\varepsilon)I + \mu S \\ kE + \mu E \\ -kE + \gamma I + \mu I \\ -\gamma I + \mu R + a\beta RI \end{bmatrix}$$

Find the Jacobian matrix of $F(x)$ and $V(x)$ evaluated at $E_0 = (N, 0, 0, 0)$, we obtained,

$$F(E_0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \beta N(1-\varepsilon) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } V(E_0) = \begin{bmatrix} \mu & 0 & \beta N(1-\varepsilon) & 0 \\ 0 & k+\mu & 0 & 0 \\ 0 & -k & \gamma+\mu & 0 \\ 0 & 0 & -\gamma & \mu \end{bmatrix}$$

Find FV^{-1} , we get

$$FV^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\beta N k (1-\varepsilon)}{(k+\mu)(\gamma+\mu)} & \frac{\beta N (1-\varepsilon)}{\gamma+\mu} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus, the spectral radius of FV^{-1} denoted by $\rho(FV^{-1})$

$$\rho(FV^{-1}) = \frac{\beta N k (1 - \varepsilon)}{(k + \mu)(\gamma + \mu)}$$

We obtained the Chickenpox reproductive number as shown,

$$R_0 = \sqrt{\frac{\beta N k (1 - \varepsilon)}{(k + \mu)(\gamma + \mu)}}$$

Stability Analysis:

In this section, we show the stability of the model at both disease free equilibrium and endemic equilibrium. First, we show that the system (1)-(4) is local asymptotically stable. The stability of this system as shown in the follow theorem.

Theorem 1:

The Chickenpox- free equilibrium of the system (1)-(4) about the equilibrium E_0 , is local asymptotically stable if $R_0 < 1$, and unstable if $R_0 > 1$.

Proof:

Since $R_0 < 1$, we have the Jacobian matrix of the system (1)-(4) at $E_0 = (N, 0, 0, 0)$ is

$$J_0 = \begin{bmatrix} -\beta(1-\varepsilon)I - \mu & 0 & a\beta R - \beta S(1-\varepsilon) & a\beta I \\ \beta(1-\varepsilon)I & -k - \mu & \beta S(1-\varepsilon) & 0 \\ 0 & k & -\gamma - \mu & 0 \\ 0 & 0 & \gamma - a\beta R & -\mu - a\beta I \end{bmatrix}$$

The eigenvalues of the Jacobian matrix J_0 are obtained by solving $\det(J_0 - \lambda I) = 0$. from this, we obtain the characteristic equation,

$$(-\mu - \lambda)^2 ((-k - \mu - \lambda)(-\gamma - \mu - \lambda) - k(\beta N(1 - \varepsilon))) = 0$$

From the characteristic equation, we see that three eigenvalues are

$$\lambda_{1,2} = -\mu < 0, \lambda_3 = \frac{-b + \sqrt{b^2 - 4c}}{2} < 0, \lambda_4 = \frac{-b - \sqrt{b^2 - 4c}}{2} < 0.$$

The fourth eigenvalue will be negative if $(\beta_h N(1 - c_w) < (\gamma + \mu + \tau)$.

Theorem 2:

The Chickenpox- present equilibrium of the system (1)-(4) about the equilibrium E_1 , is local asymptotically stable if $R_0 > 1$, and unstable if $R_0 < 1$.

Proof:

Since $R_0 > 1$, we have the Jacobian matrix of the system (1)-(4) at $E_1 = (S^*, E^*, I^*, R^*)$ is

$$J_1 = \begin{bmatrix} -\beta(1-\varepsilon)I - \mu & 0 & a\beta R - \beta S(1-\varepsilon) & a\beta I \\ \beta(1-\varepsilon)I & -k - \mu & \beta S(1-\varepsilon) & 0 \\ 0 & k & -\gamma - \mu & 0 \\ 0 & 0 & \gamma - a\beta R & -\mu - a\beta I \end{bmatrix}$$

Where S^*, I^*, R^*, B^* are given by equation (4). The characteristic equation of Jacobian matrix at E_1 , given by equations (1)-(4), becomes

$$(-\beta(1-\varepsilon)I^* - \mu - \lambda)((-\mu - a\beta I^* - \lambda)((-k - \mu - \lambda)(-\gamma - \mu - \lambda) - k(\beta S^*(1 - \varepsilon)))) \\ + (\beta(1-\varepsilon)I^*)(k((a\beta R^* - \beta S^*(1 - \varepsilon))(-\mu - a\beta I^* - \lambda) - (\gamma - a\beta R^*)(a\beta I^*))) = 0$$

We obtain, the eigenvalues are

$$\lambda^4 + w\lambda^3 + x\lambda^2 + y\lambda + z.$$

where

$$\begin{aligned}
 w &= ((\beta(1-\varepsilon)I^* + \mu) + (\mu + a\beta I^*) + (k + \mu) + (\gamma + \mu)) \\
 x &= (\beta(1-\varepsilon)I^* + \mu)((\mu + a\beta I^*) - (k + \mu) - (\gamma + \mu)) - (\mu + a\beta I^*)(-(k + \mu) - (\gamma + \mu)) \\
 &\quad + (k + \mu)(\gamma + \mu) - (k(\beta S^*(1-\varepsilon))) \\
 y &= (\beta(1-\varepsilon)I^* + \mu)(k + \mu)(\gamma + \mu) + (\beta(1-\varepsilon)I^* + \mu)(k(\beta S^*(1-\varepsilon))) \\
 &\quad - (\mu + a\beta I^*)(-(k + \mu) - (\gamma + \mu)) - (\mu + a\beta I^*)(k + \mu)(\gamma + \mu) - k(\beta S^*(1-\varepsilon)) \\
 &\quad + (\beta(1-\varepsilon)I^*)(k)(a\beta R^* - \beta S^*(1-\varepsilon)) \\
 z &= ((k + \mu)(\gamma + \mu) - k(\beta S^*(1-\varepsilon)))(\beta(1-\varepsilon)I^* + \mu)(\mu + a\beta I^*) \\
 &\quad - (\beta(1-\varepsilon)I^*)(k)(a\beta R^* - \beta S^*(1-\varepsilon))(-\mu - a\beta I^* - \lambda) + (\beta(1-\varepsilon)I^*)(\gamma - a\beta R^*)(a\beta I^*)
 \end{aligned}$$

Since all eigenvalues have negative real part which they satisfy the Routh-Hurwitz conditions (Marsden and McCracken, 1976).

Numerical Results:

The parameters used in the numerical simulation results are given in Table.1

Table 1: Parameter values in numerical simulations at Chickenpox - Free State.

Parameters	Descriptions	Values
λ	Birth rate	0.0000391 per day
μ	Natural death rate of individuals	0.0000391 per day
β	Contact rate	0.00026
k	Incubation period	0.036 per day
γ	Recovery rate	0.16 per day
a	Reinfected rate	0.021 per day
ε	Effectiveness of individuals sanitation	0.99999
N	Total number of population	2000 person

Stability of Chickenpox- Free State:

Using the values of parameters as shown in Table.1. We obtained the eigenvalues and the Chickenpox reproductive number as follows;

$$\lambda_1 = -0.160041, \lambda_2 = -0.0360376, \lambda_3 = -0.0000391, \lambda_4 = -0.0000391$$

$$R_0 = 0.0000324568.$$

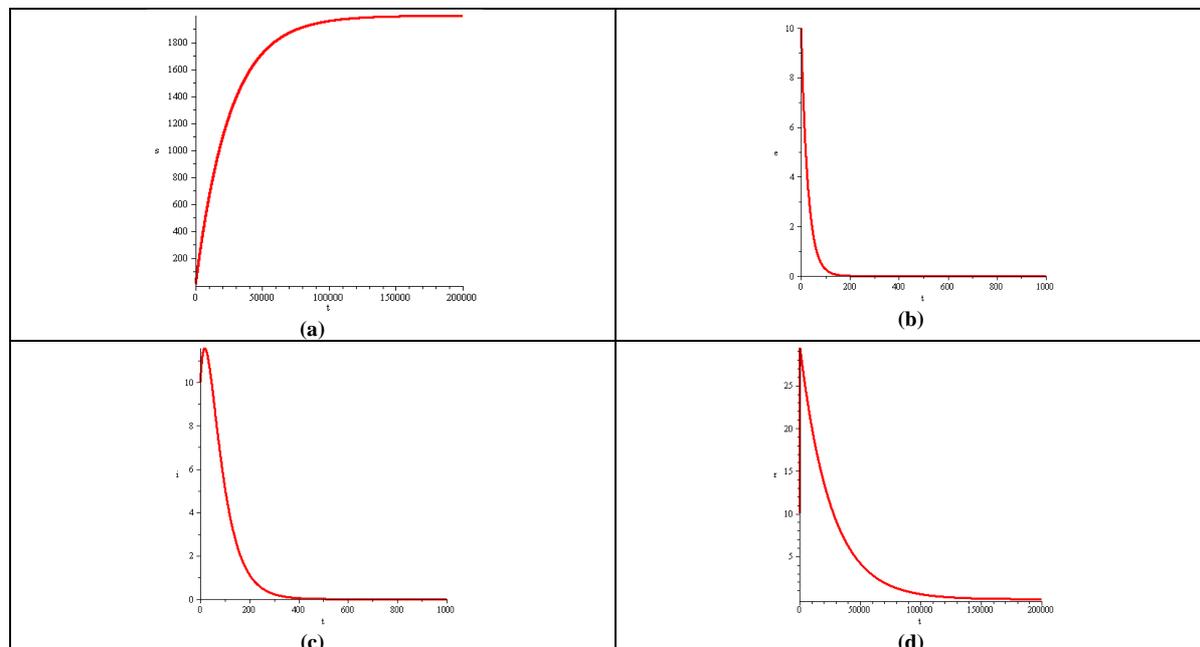


Fig. 2: Time series of (a) Susceptible population (S), (b) Exposed population (E), (c) Infectious population (I) and (d) Recovered population (R) with the values of parameters; $N = 2000$, $\lambda = 0.0000391$, $\beta = 0.00026$, $k = 0.036$, $\gamma = 0.16$, $\mu = 0.0000391$, $a = 0.021$, $\varepsilon = 0.99999$.

We see that the solutions approach to the Chickenpox- free equilibrium E_0 (2000, 0, 0, 0).

Stability of Chickenpox present state:

We change the value of the effectiveness of individual sanitation to $\mathcal{E} = 0.6785$, $\gamma = 0.012$, $\beta = 0.00029$ and keep the other values of parameters to be those given in Table.1. We obtain the eigenvalues and the Chickenpox reproductive number as follows;

$$\lambda_1 = -0.156082, \lambda_2 = -0.0000490937 - 0.000773306i, \lambda_3 = -0.0000490937 + 0.000773306i, \lambda_4 = -1.86069 \times 10^{-8}, R_0 = 1.55173$$

Since all eigenvalues are to be negative and the chickenpox reproductive number is greater than one, the Chickenpox -present equilibrium state, E_1 will be local asymptotically stable as shown in Fig. 3 and Fig. 4.

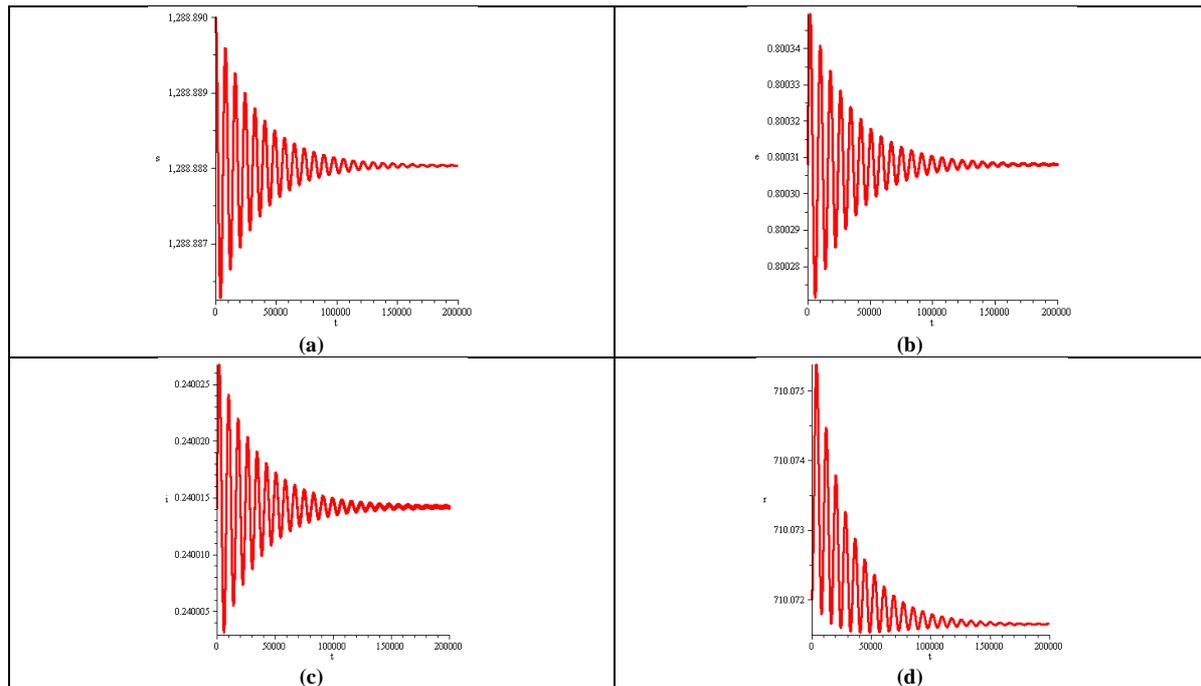


Fig. 3: Time series of (a) Susceptible population, (b) Exposed population , (c) Infectious population and (d) Recovered population with the values of parameters; $N = 2000, \lambda = 0.0000391, \beta = 0.00029, k = 0.036, \gamma = 0.12, \mu = 0.0000391, a = 0.021, \mathcal{E} = 0.6785$. The state variables approach to Chickenpox-present equilibrium $E_1 = (1288.89, 0.800308, 0.240014, 710.072)$.

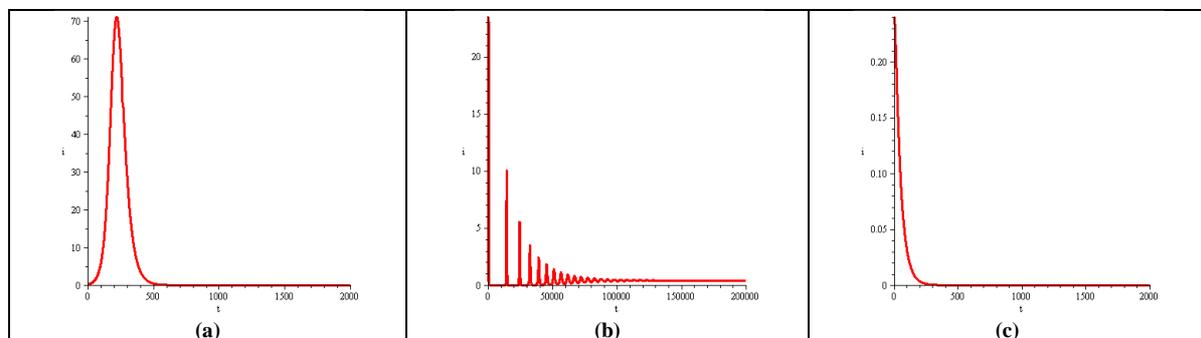


Fig. 4: Time series of Infectious population (a) $\epsilon = 0.2$, (b) $\epsilon = 0.5$ and (c) $\epsilon = 0.9$. $N = 2000, \lambda = 0.0000391, \beta = 0.00029, k = 0.036, \gamma = 0.12, \mu = 0.0000391, a = 0.021$, The state variables approach to Chickenpox-present equilibrium.

Conclusion:

In this study, we proposed the mathematical model of chickenpox with the effect of individuals' sanitation and analyzed the analytical results by stability theory of differential equation and computer simulation. The Chickenpox reproductive number is obtained through the use of spectral radius of the next generation matrix.

The Chickenpox reproductive number is $R_0 = \sqrt{\frac{\beta N k (1 - \varepsilon)}{(k + \mu)(\gamma + \mu)}}$. The Chickenpox reproductive number is

the threshold condition for determining the stability of the equilibrium points of the model which are shown in Fig. 2, Fig. 3 and Fig. 4. Our simulation results shown that R_0 will be decrease when the effectiveness of individuals' sanitation is increased. We found the value of R_0 was 1.56021, 1.27391, 0.900789, 0.180158 when $\varepsilon = 0.25, 0.50, 0.75, 0.99$ respectively. It seen that the infected human will decrease if the infectious human be awareness about the infectious waste to contribute in the environment.

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